A Statistical Study of Selective Fading of Super-High Frequency Radio Signals

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The results of two months of comprehensive frequency-sweep measurements of selective fading in the band between 3750 and 4190 mc over a radio relay path in Iowa are reported. An abridgement of the data, general conclusions derived from the data and an example of the use of the data in connection with frequency diversity measures for radio relay systems are given.

INTRODUCTION

It is well known that, in the high-frequency range during fading conditions, radio signals on different frequencies may exhibit at any instant radically different behavior. This may be true even though they are in the same frequency band and exhibit the same statistical behavior, when observed over a longer period of time. It is also known that h-f fading may be frequency selective enough within the narrow limits of a single radio channel to cause severe distortion of modulated signals. It has been established that the cause of these phenomena is multipath transmission. This knowledge, which is of long standing in the high-frequency range, raised questions concerning the prevalence of similar phenomena in the super-high-frequency range about which relatively little has been known until recently.

During recent years studies of super-high-frequency propagation and fading have been made which have been previously reported.[1] In these tests a frequency-sweep method was used to determine how the loss of a particular radio path varied with frequency at a given instant; and short-pulse methods were used to determine the path length differences which were involved when multipath transmission occurred. These are

[1] A. B. Crawford and W. C. Jakes, Jr., Selective Fading of Microwaves, and O. E. DeLange, Propagation Studies at Microwave Frequencies by Means of Very Short Pulses, Bell System Tech J., **31**, Jan., 1952.

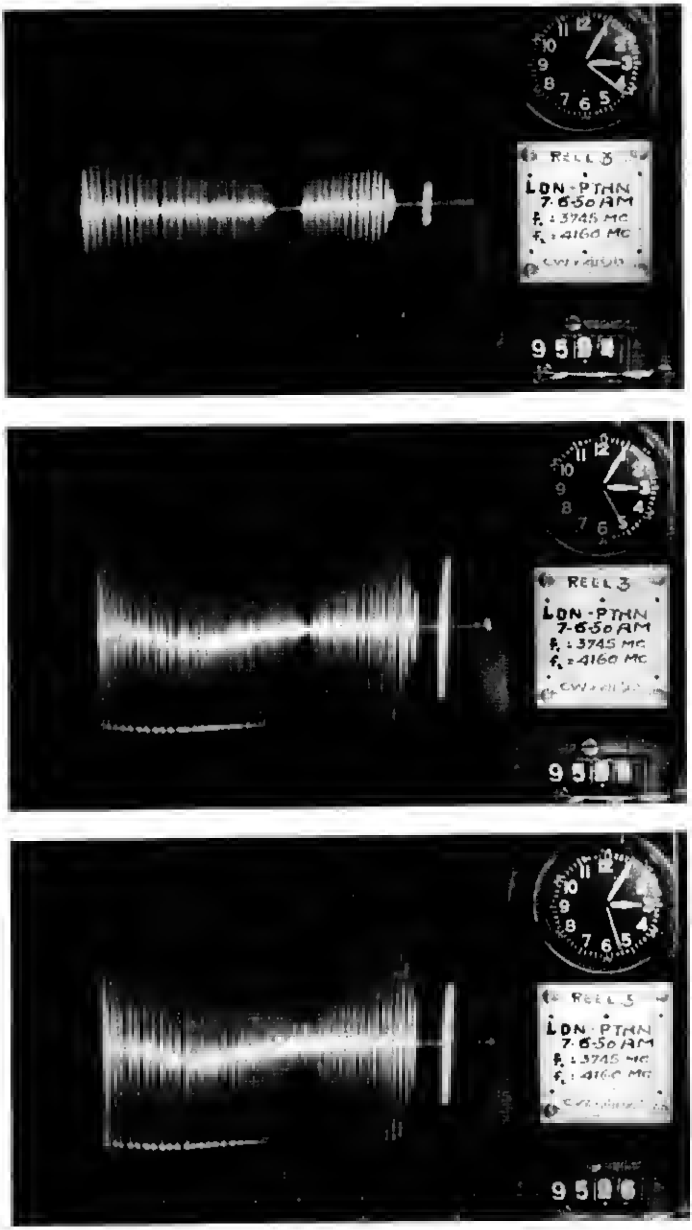

Fig. 1 — Typical records obtained during the Lowden-Princeton, Iowa, frequency-sweep transmission measurements. The horizontal scale is proportional to frequencies between 3,750 and 4,150 mc (the single spike on the right is at 4,190 mc); and the vertical scale is proportional to the amplitude of the received signal.

standard methods for studying multipath transmission (or selective fading) and the results obtained were in good agreement.

Some frequency-sweep transmission measurements were made during the summer of 1950 over a typical radio relay path in the mid-continent region of the United States for the purpose of obtaining additional data of a statistical nature which could be used in system engineering.

The objective of the tests was to determine what per cent of the fading was frequency selective and the degree of its frequency selectivity. Such data were needed to evaluate the advantages of using frequency diversity as a means of minimizing the effects of fading, and to provide quantitative data for designing frequency-diversity measures. Recordings were made throughout the months of July and August which serve as the basis for a statistical picture of the fading occurring during those months in the frequency band between 3,750 and 4,190 mc.

Path Over Which Measurements Were Made

The measurements were made over a 30.8-mile path between the Lowden and Princeton (Iowa) towers of the Chicago-Omaha Radio Relay System. This path was chosen as a typical radio relay path as to length, clearance above terrain, and climatic conditions. Height-loss runs made by the American Telephone and Telegraph Company (when originally selecting the path as part of their radio relay route) indicated that ground reflections on this path were unimportant. The reflection coefficient was less than 0.1

Method of Tests

The frequency-sweep equipment used in these tests was similar to that used in the previous studies.[1] With this equipment about 50,000 record photographs were obtained of a cathode-ray tube presentation of the path-loss vs frequency characteristic of the path between 3,750 and 4,150 mc. Fig. 1 shows three illustrations of these record photographs, which are more fully discussed below. The taking of these records was distributed throughout the months of July and August in such a manner as to give complete coverage of all the fading during that period. In addition the single-frequency path loss at 4,190 mc was continuously recorded throughout the two months.

The data from these records were analyzed on a statistical basis; and families of curves were obtained depicting several aspects of the nature of the selective fading encountered during the tests. These are described more fully below.

GENERAL DISCUSSION OF FADING PHENOMENA

The variations in the strength of a received radio signal known as "fading" are caused by variable or temporary conditions in the transmission path. These conditions fall into two broad classes: those causing partial obstruction of the path, and those causing multipath transmission. The latter class is believed to be the principal source of frequency-selective fading.

Multipath transmission involves the reception of more than one signal ray, each of which travels over a different path between the transmitter and receiver. Generally each such path has a different length between the transmitter and receiver. Multipath transmission involves either or both reflection or refraction of at least one (and in some cases all) of the received rays. Since the conditions of the atmosphere are continuously varying, the paths of the received rays are variable with time. The received signal is the resultant of all the rays accepted by the receiving antenna. The relative phase of the different received rays depends on (1) the differences in the lengths of the paths over which they have travelled and (2) the signal frequency. If the rays arrive at the receiving antenna in nearly the same phase, they add and enhance the received signal; if they arrive in phase opposition they partially cancel each other and fading results. This fading is not only variable with time but also with signal frequency, and is called "(frequency) selective fading". Since the conditions which cause this kind of fading are substantially random, the variation of fading with time on a statistical basis might be expected to approach the Rayleigh distribution. There has been experimental confirmation of this.

A number of other factors are involved which will not be treated here, since the mechanism and effects of multipath transmission have been discussed quite thoroughly in the previously mentioned reports by Crawford and Jakes, DeLange and in an earlier paper.[2]

Results of Tests

Fig. 1 shows three illustrations of the type of records obtained. On each record the horizontal deflection of the cathode-ray-tube trace is proportional to the radio signal frequency. The vertical deflection is linearly proportional to the amplitude of the received signal, which because of constant transmitter power is inversely proportional to the

[2] H. T. Friis, Microwave Repeater Research, Bell System Tech. J., **27**, Apr., 1948.

path loss.[3] In each record one second was required for the trace to travel through the entire frequency range covered.

The three records shown are consecutive, being taken at 3:05:21 AM, 3:05:25 AM and 3:05:27 AM. The first record (frame number 9524) was taken with 10 db more attenuation in the input to the measuring equipment than when the later two records were taken (frame numbers 9525 and 9526). There is obvious overloading in the left hand portions of the records on frames 9525 and 9526; but the accuracy of the right hand portions of these records (showing the deeper part of the fade) is unimpaired. The noticeable difference between the shapes of the curves near the deeper parts of the fade on frames 9525 and 9526 is typical. These changes occurred within two seconds. Generally, the deeper parts of the fades show more rapid changes than the less deep parts.

From the 50,000 record photographs all those pertaining to fading of 30 db or more were segregated and analyzed. The remaining records were analyzed on a sampling basis, except that every record showing unique effects was analyzed. About 1,800 path-loss versus frequency curves such as those illustrated in Figs. 2 and 3 were obtained. These curves were separately studied, and were also treated statistically.

Because the levels during the deeper part of the fade are too low to show on frame 9524, and because portions of frame 9525 showing the less severe part of the fade are affected by overloading, it was necessary to combine two records (shown on Fig. 1) to obtain the single path-loss versus frequency curve shown as Fig. 2(a).

Theoretically, it should be possible to synthesize by means of an addition-of-vectors method each of the path-loss versus frequency curves obtained from these tests. Each vector term of the equation would correspond to a component of the received signal and would be of the form $R \cos \omega T$, where: R is the magnitude of a particular component (normalized to the magnitude of the direct signal component), T is the delay (in seconds) between the time of arrival of the direct signal component and the particular interfering component, and ω is 2π times the frequency.

In practice, however, it has been found in the case of deep fades that components of relatively small magnitude and relatively long delay are of importance in determining the shapes of the curves near maxima of path loss. Also it has been found that in most such cases quite a few components are involved. These factors make accurate analysis of many

[3] A logarithmic amplitude characteristic (db scale) would have been preferable; but time did not permit modifications of the test equipment before the start of the 1950 fading season.

curves impractical without a computer capable of handling a relatively large number of components several of which may be relatively small.

However, experimentation with a trial and error method of solution on a few selected curves has shed considerable light on the nature of received signals which could produce the types of curves included in the data under consideration.

Curve (a) on Fig. 4 is one of those selected for analysis and was synthesized by using a direct signal component and six interfering components as shown in Table I. If components 5 and 6 had been omitted from the synthesis, curve (b) would have resulted; and, if components, 3, 4, 5 and 6 had been omitted curve (c) would have been obtained.

The difference between curves (a) and (b) illustrate the radical effect which can be produced on the shape of a curve near maxima of path loss

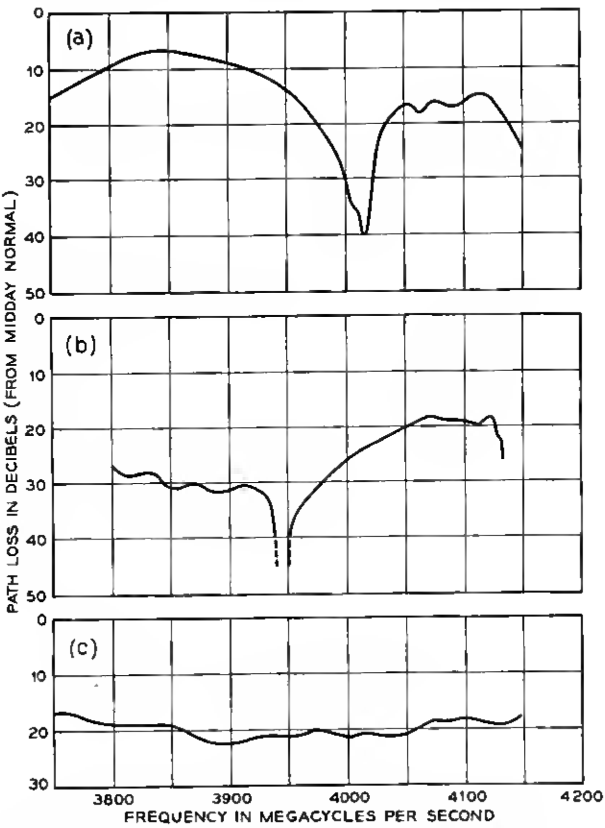

Fig. 2 — Typical path-loss versus frequency curves observed on the Lowden-Princeton, Iowa, path during July and August, 1950.

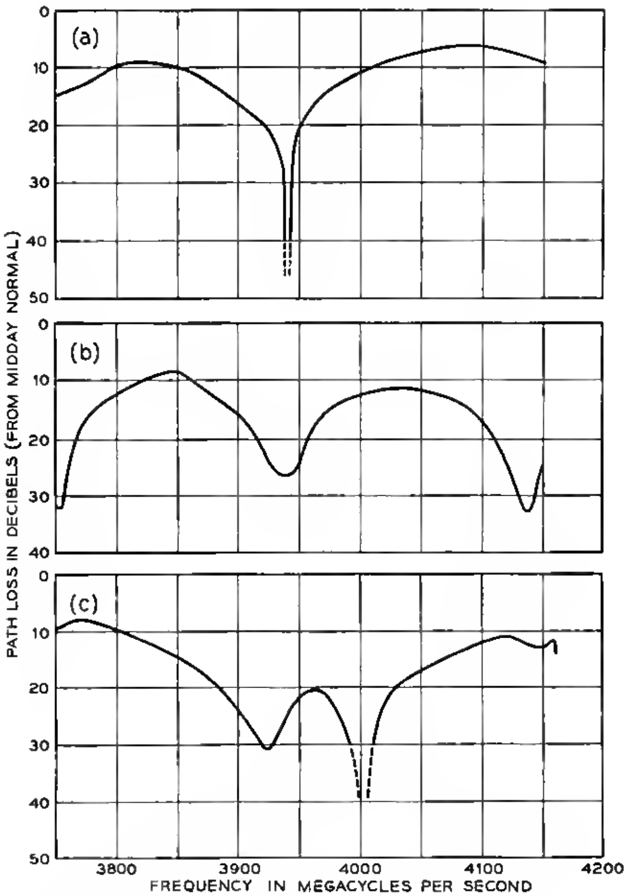

Fig. 3 — Typical path-loss versus frequency curves observed on the Princeton-Lowden, Iowa, path during July and August, 1950.

by relatively small components such as components 5 and 6. In the case of deeper fades much smaller components may be of importance in determining the shapes of the curves near maxima of path loss; hence the curves for deep fades are quite difficult to synthesize. The shape near the maximum of path-loss on the curve in Fig. 2(a) is quite typical of the shapes to be found in the data under discussion.

Based on the experience gained in attempting to synthesize some of the curves, it is possible to recognize the significance of certain features of other curves, and to generalize the nature of all the curves. The principal conclusions are:

1. No deep fades were found which did not show definite frequency

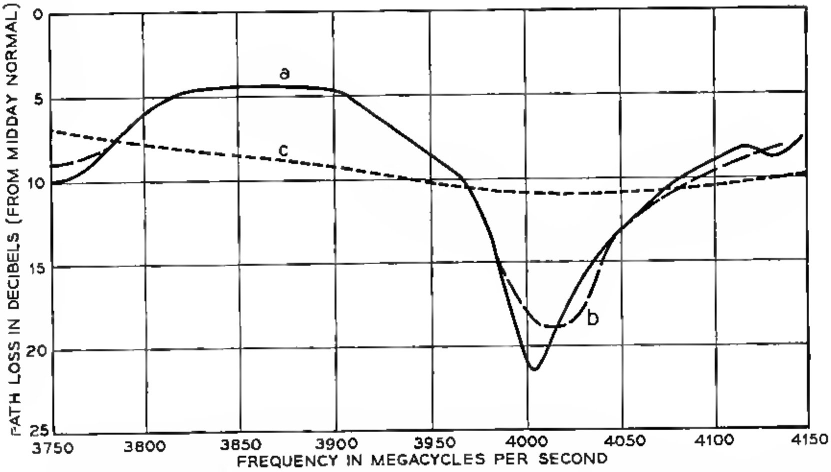

Fig. 4. — Path-loss versus frequency curves synthesized from components shown in Tablo I.

selectivity. (Statistical data are given later whicb bear out this conclusion).

2. No deep fades were found that did not include an appreciable component of loss with little appearance of frequency selectivity; that is, no deep fading occurred unless the signal was already depressed 10 or so db across tbe entire observed band. This could be caused eitber by (a) the presence of appreciable interfering signals of slight delay from the direct signal component, see curve (c) on Fig. 4, or (b) by non-frequency selective attenuation of the direct signal component. The former seems more likely; but the evidence is not conclusive in this regard.

3. No curves for deep fades were found which appeared as if they could be synthesized satisfactorily with fewer than four to six components.

TABLE I

Component	R (Normalized to magnitude of direct received signal)	T Milli-Microseconds	Phase Shift From Direct Signal Component	
			Half Wave Lengths	At Frequency
0	1.0	0	0	Any Freq.
1	0.45	0.122	1	4090 mc
2	0.26	0.370	3	4050 mc
3	0.10	2.86	23	4033 mc
4	0.115	3.14	25	3981 mc
5	0.025	3.9	31	3972 mc
6	0.02	12.1	97	3993 mc

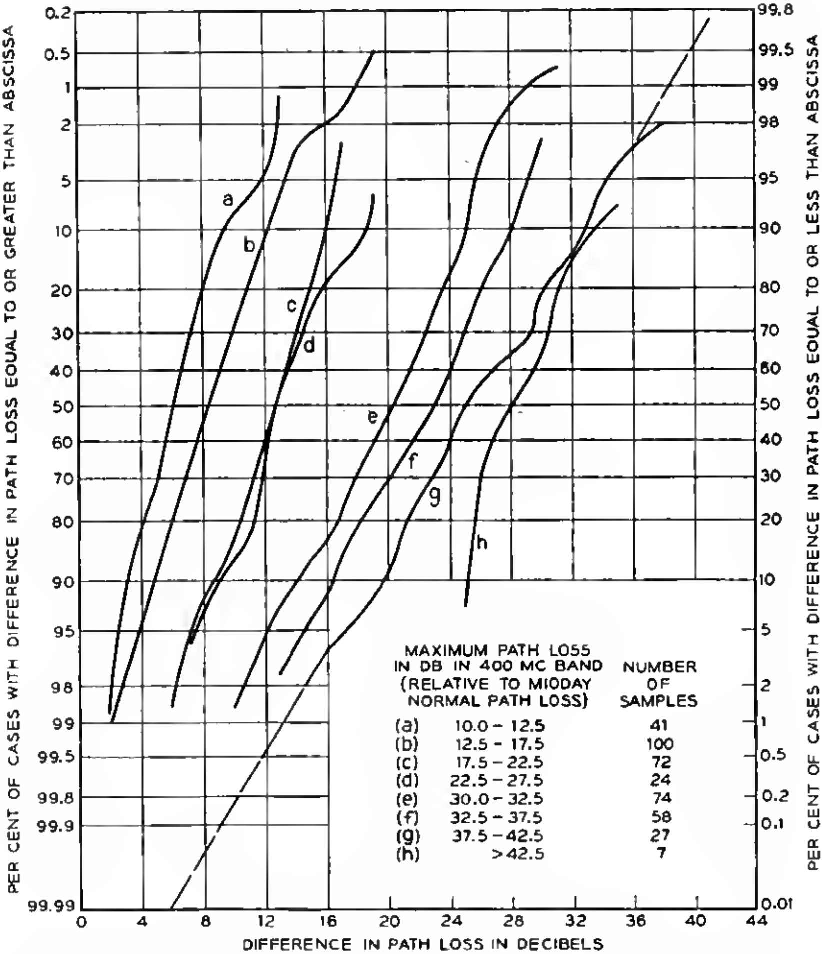

Fig. 5 — Statistical distribution of differences between maximum and minimum path loss in bands 400 mc wide. (Center frequency of 400-mc band random with respect to frequencies corresponding to maxima and minima of path loss).

4. It seemed to be the rule that components with relatively long delays were appreciably smaller in magnitude than those with shorter delays.

5. At the bottom of the deep fades some of the small signal components with relatively long delays were an important factor in determining the shape of the path-loss versus frequency curves.

Fig. 5 illustrates one type of statistical information which was derived from the 1,800 individual path-loss versus frequency curves. The family of curves on Fig. 5 shows the per cent of fades (of a given depth) which

were frequency selective to any given degree. For example, 95 per cent of fades which were 40 db deep showed a variation in path loss (frequency selectivity) across the observed band 400 mc wide of at least 17 db. By extrapolating the curve for 40 db fades (whicb appears to follow a normal distribution law) it can be estimated that 99 per cent of the 40 db fades had at least 13 db of frequency selectivity within the ohserved band 400 mc wide. Probably, observation of a wider band would have shown even greater evidence of frequency selectivity.

During periods when the received signal was materially stronger than mid-day normal there was practically no evidence of frequency selectivity. Several periods were observed when the path-loss across the entire

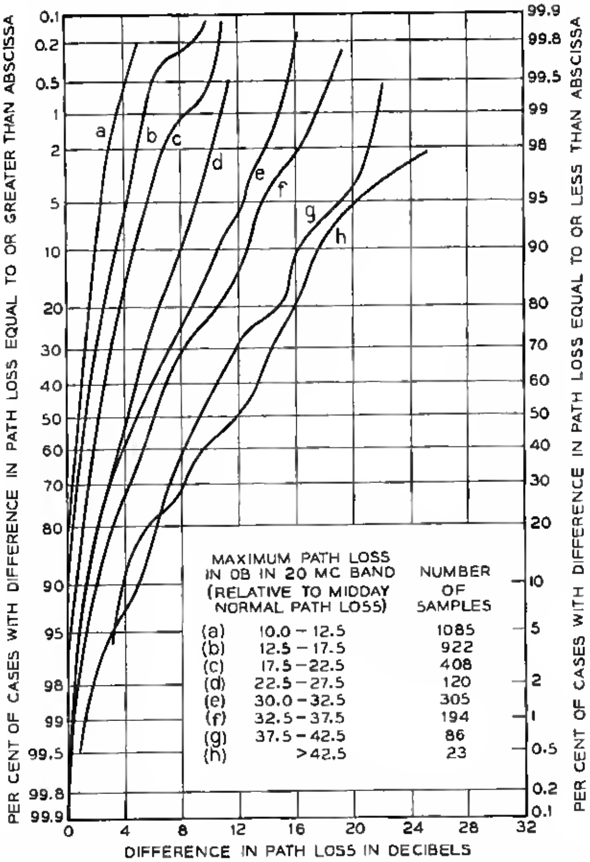

Fig. 6 — Statistical distribution of differences between maximum and minimum path loss in bands 20 mc wide. (Center frequency of 20-mc band random with respect to frequencies corresponding to maxima and minima of path loss).

400-mc observed band was 8 db less than mid-day normal; and in one case it was more than 10 db less than mid-day normal.

Another family of curves is shown by Fig. 6. These curves were obtained by dividing the 400-mc observed band into a number of 20-mc bands, chosen at random with regard to the shapes of the path-loss versus frequency curves. These data show the difference between the maximum and minimum losses within a single 20-mc broad-band channel which might accompany a fade of a given depth. Such data are of use in estimating possible distortions of a modulated signal occupying a band width of 20 mc.

Frequency Diversity

The fact that the instantaneous fading may be different on different frequencies within the same frequency range offers a means for mitigating transmission impairments caused by fading. During periods when there is fading in excess of a specified value on the regular carrier frequency, the carrier can be shifted to an alternate frequency in the hope that the fading on the alternate frequency may be less severe. The merit of using this type of frequency diversity can be gauged from the statistical distribution of fading on the alternate frequency during periods when there is fading in excess of the specified value on the regular frequency.

The data obtained in these tests indicates that there was no correlation between the fading on frequencies separated by: (1) 40 mc or more during periods when there was fading of 10 db on one of the frequencies and (2) 160 mc or more during periods when there was fading of at least 20 db on one of the frequencies. However, during periods of severe fading, the data indicate considerable correlation between the fading on regular and alternate frequencies separated by 80 mc or less.

Fig. 7 shows the distributions of fading on alternate frequencies at specified frequency separations from the regular frequency, when there is fading of a specified depth on the regular frequency. Table II indicates which of the curves on Fig. 7 applies to a particular set of conditions.

Curves G and H on Fig. 8 show the distributions of depths of fade at 4190 mc during the entire months of July and August, respectively. Comparison of the 4,190-mc data with data from tests made over other paths indicates that the fading occurring during the Iowa tests was nearly as severe as during the "worst month" observed on any path to date.

Curve B on Fig. 7 shows the statistical distribution of fading on any frequency (within the range under consideration) during periods when important fading is prevalent. The fading shown by this curve is con-

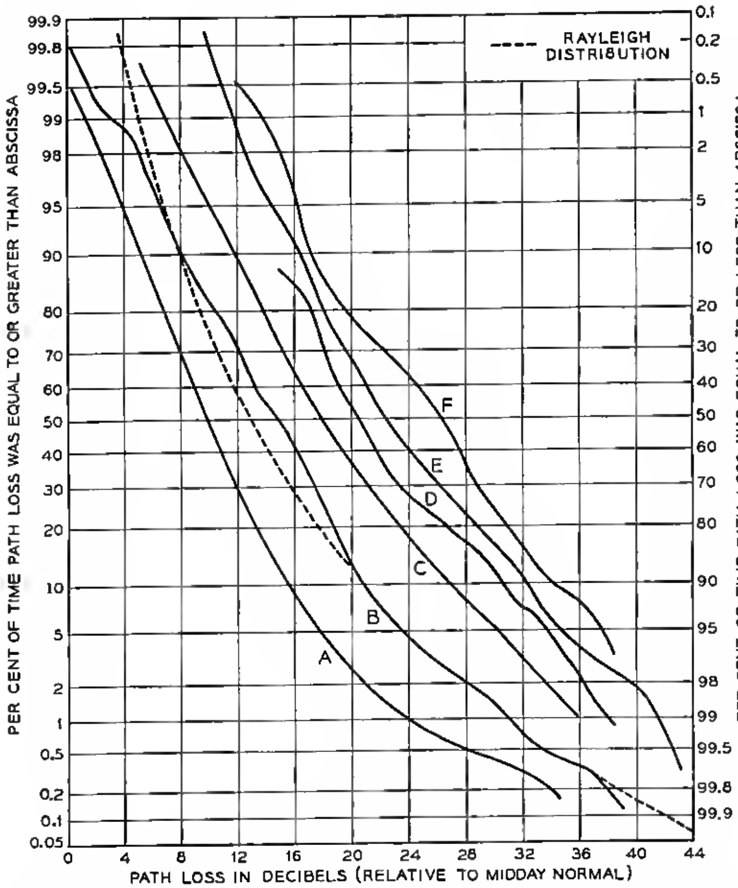

Fig. 7 — Statistical distributions of fading measured on Lowden-Princeton, Iowa, path during July and August, 1950. These curves are useful in predicting the efficacy of specific frequency-diversity systems as described in the text.

siderably more severe than is shown by the curves on Fig. 8. This is because the latter curves include all the time within the months under consideration. Therefore, they include much time when the fading mechanism was not present in the path, and the path loss remained steady near mid-day normal.

There is evident similarity between the shapes of Curves A through F (on Fig. 7) with the shape of the curve which is based upon the Rayleigh distribution. This is consistent with the theory that deep fading associated with multipath transmission is caused by random phasing of a large number of vectorial components.

These data have been applied practically to the design of frequency-diversity measures for minimizing circuit outages caused by fading in TD-2 radio relay systems.

Fig. 9 shows an illustration of the practical use of the data. Curve *H* (which is the same as Curve *H* on Fig. 8) shows the distribution with time of the fading on a typical radio relay path without frequency diversity measures. Curve *J* shows the distributions of fading if certain specific frequency diversity measures are used (based solely on fading considerations). The difference between Curves *H* and *J* shows the improvement gained by using that specific kind of frequency diversity. For example, Curve *H* shows that without frequency diversity fading in excess of 30 db will occur 0.075 per cent of the time and fading in excess of 40 db will occur 0.02 per cent of the time. But, if the kind of frequency diversity to which Curve *J* corresponds is used, fades deeper than 30 db will occur only 0.0012 per cent of the time, and fades deeper than 40 db will occur only 0.00008 per cent of the time. Thus the improvement resulting from this type of frequency diversity is a reduction of fades deeper than 30 db from 0.075 to 0.0012 per cent of the time, and fades deeper than 40 db from 0.02 to 0.00008 per cent of the time.

Table II

Depth of Fade on Regular Frequency	Separation Between Regular and Alternate Frequencies	Curve
10 db	40 mc or more	A
20	40 mc	C
20	80 mc or more	B
30	40 mc	E
30	80 mc	C
30	160 mc or more	B
40	40 mc	F
40	80	D
40	160 mc or more	B

To explain how Curve *J* was derived, let us assume that during any time when there is a fade of 30 db or more on the operating frequency of a given channel the signal will be switched to another channel frequency. Let us further assume that the alternate frequency will be separated by 160, 240, 320, or 400 mc from the assumed operating frequency and that the choice of alternate frequency is random. If we also assume that the conditions of August, 1950, on the Iowa path prevail, we will find from Curve *H* on Fig. 8 that the assumed operating frequency will have a fade of 30 db or more 0.075 per cent of the time. Then a new distribution curve can be prepared, based on (1) Curve *B* (on Fig. 7) for

0.075 per cent of the time, and (2) that portion of Curve H (on Fig. 8) which corresponds to fades of less than 30 db for the remaining 99.925 per cent of the time. Curve J on Fig. 9 is such a curve.

If frequencies separated from the assumed operating frequency by 80 mc were used instead of those assumed above, Curve C instead of Curve B on Fig. 7 would have been used. This would have shown a somewhat smaller improvement because of some correlation between the fading on the assumed operating frequency and an alternate frequency separated from it by 80 mc. However, the improvement gained from the use of this system would be ample to prove-in its use.

The question is sometimes raised concerning the reason that the curves on Fig. 9 show an apparent improvement for fading of less than 30 db, since the circuit is switched only when fading deeper than 30 db occurs. This is because the curves are cumulative distribution curves;

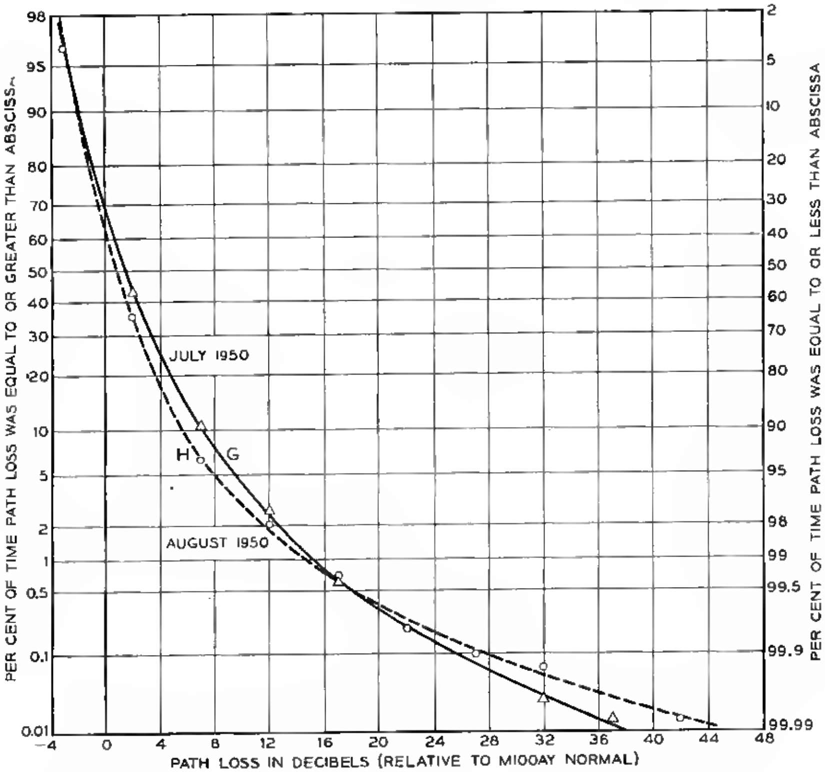

Fig. 8 — Statistical distribution of fading loss Princeton-Lowden, Iowa, path July and August, 1950.

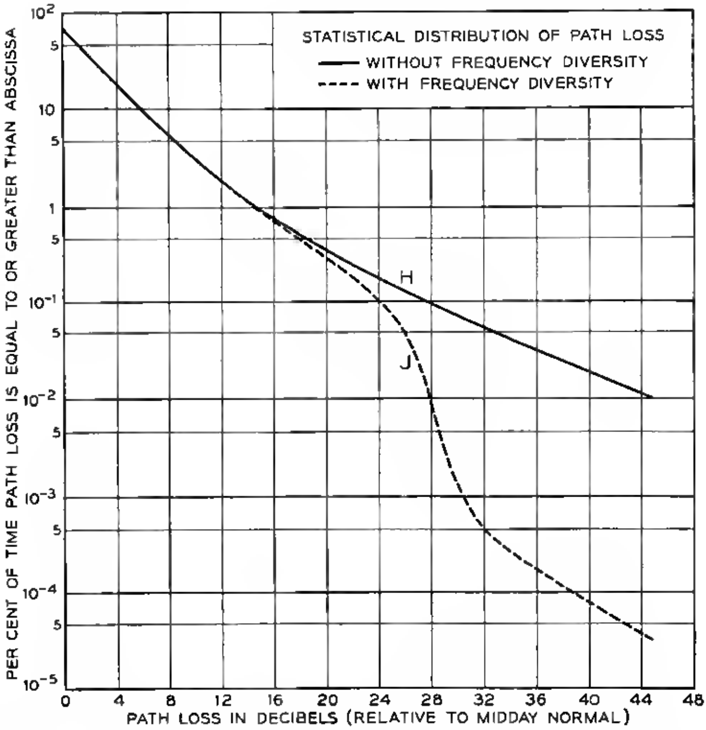

Fig. 9 — Effect on transmission of use of frequency diversity in 3,750–4,150 mc range.

and reduction of the per cent of time when there is fading deeper than 30 db also reduces the per cent of time when there is fading deeper than 25 db, etc.

CONCLUSIONS

Quantitative information on fading phenomena is essential in the engineering of super-high-frequency systems and in evaluation of frequency diversity arrangements. The data obtained from the field tests and the statistical analysis reported herein, while limited to a single path and particular season, fill a gap in previous knowledge. They have found practical application in the design of diversity systems for improving the reliability of radio relay systems.

The principal conclusions that can be drawn from the available data are: (1) all of the deep fading is definitely frequency selective, and is caused by a complex multi-path transmission; (2) deep selective fading

is ordinarily accompanied by a 6 to 10 db signal depression over a band of at least several hundred megacycles; (3) fading on frequencies separated by 160 or more megacycles shows little correlation, but fading on frequencies more closely spaced shows an increasing correlation as the frequency difference is diminished; and (4) frequency diversity offers a practical means of mitigating circuit impairment due to fading, if the transmission can be shifted to a frequency far enough away to minimize correlation with fading on the original frequency.

ACKNOWLEDGEMENTS

The author wishes to acknowledge the important contribution of J. Mallett, who conducted the field tests during which the data were obtained. The field tests and data analysis were under the supervision of R. P. Booth.